A Simulation and Optimization Methodology for Reliability of Vehicle Fleets

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2011-01-0725

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **13 APR 2011**	2. REPORT TYPE **Briefing Charts**	3. DATES COVERED **13-04-2011 to 13-04-2011**

4. TITLE AND SUBTITLE **A SIMULATION AND OPTIMIZATION METHODOLOGY FOR RELIABILITY OF VEHICLE FLEETS**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) **Zissimos Mourelatos; Jing Li; Matt Castanier; Amandeep Singh; David Lamb**	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **U.S. Army TARDEC ,6501 E.11 Mile Rd,Warren,MI,48397-5000**	8. PERFORMING ORGANIZATION REPORT NUMBER **#21740**
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **U.S. Army TARDEC, 6501 E.11 Mile Rd, Warren, MI, 48397-5000**	10. SPONSOR/MONITOR'S ACRONYM(S) **TARDEC** 11. SPONSOR/MONITOR'S REPORT NUMBER(S) **#21740**

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES
Submitted to 2011 SAE WORLD CONGRESS

14. ABSTRACT
Briefing charts.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**43**	

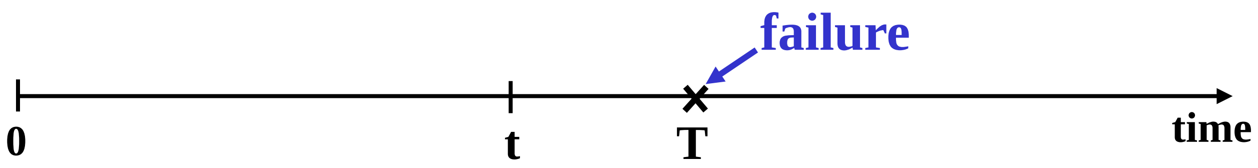
Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

Overview

- **What is reliability ??**
- **Basics of reliability methods for repairable and non-repairable systems**
- **Estimation of PDF of Time Between Failures (TBF) using limited, censored data**
  - **"Frequentist" approach (Method 1)**
  - **Bayesian updating approach (Method 2)**
    - ✓ **"Enhances" data with expert opinion**

What is Reliability?

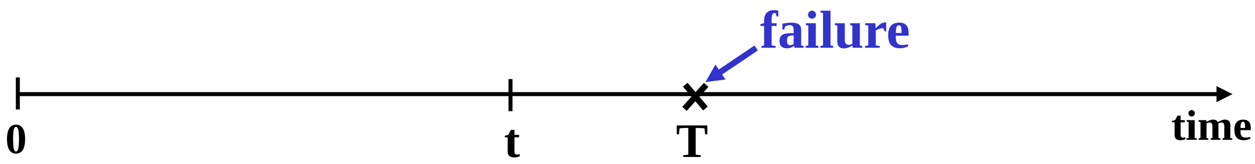
Reliability at time t is the probability that the system has not failed before time t.



$$R(t) = P(T > t) = 1 - P(T \leq t)$$

Reliability Basics for Non-Repairable Systems

Reliability of Non-Repairable Systems



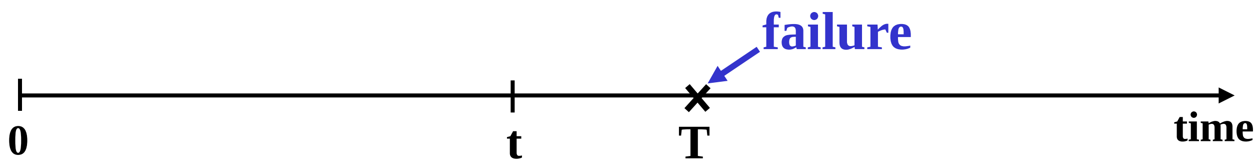
$$R(t) = P(T > t) = 1 - P(T \le t) \Rightarrow \boxed{R(t) = 1 - F(t)} \qquad \textbf{(1)}$$

$$\lambda(t) = \frac{P(t < T \le t + dt / T > t)}{dt} = \frac{P(t < T \le t + dt)}{dt * P(T > t)} =$$

Failure Rate

$$= \frac{F(t + dt) - F(t)}{dt * R(t)} \Rightarrow \boxed{\lambda(t) = \frac{f(t)}{R(t)}} \qquad \textbf{(2)}$$

Reliability of Non-Repairable Systems



due to (2)

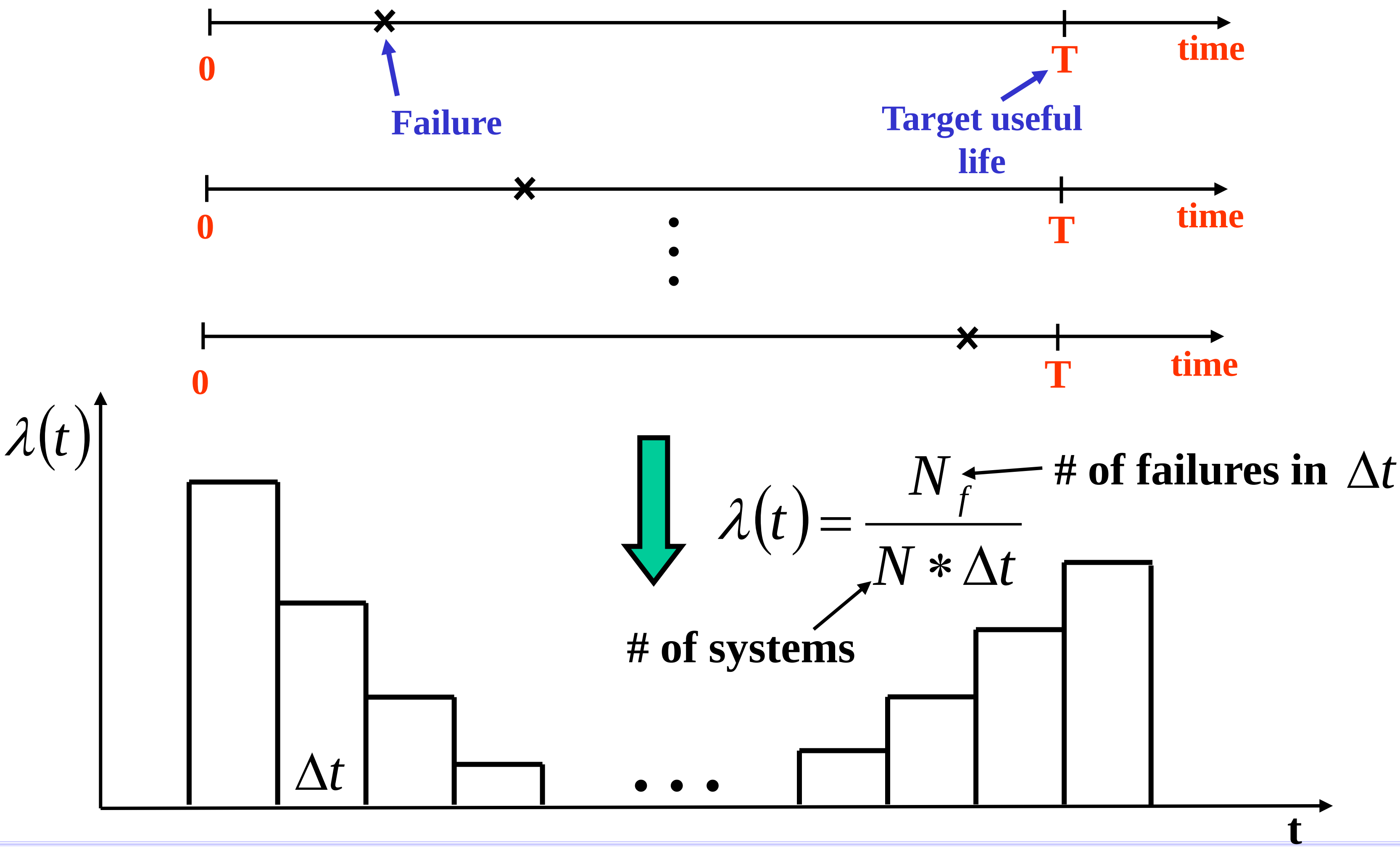
$$R(t) = 1 - F(t) \Rightarrow \frac{dR}{dt} = -f(t) \Rightarrow \frac{dR}{dt} = -\lambda(t)R(t) \Rightarrow$$

$$\Rightarrow \frac{dR}{R} = -\lambda dt \Rightarrow d(\ln R) = -\lambda dt \Rightarrow \ln\left(\frac{R(t)}{\cancelto{1}{R(0)}}\right) = -\int_0^t \lambda dt \Rightarrow$$

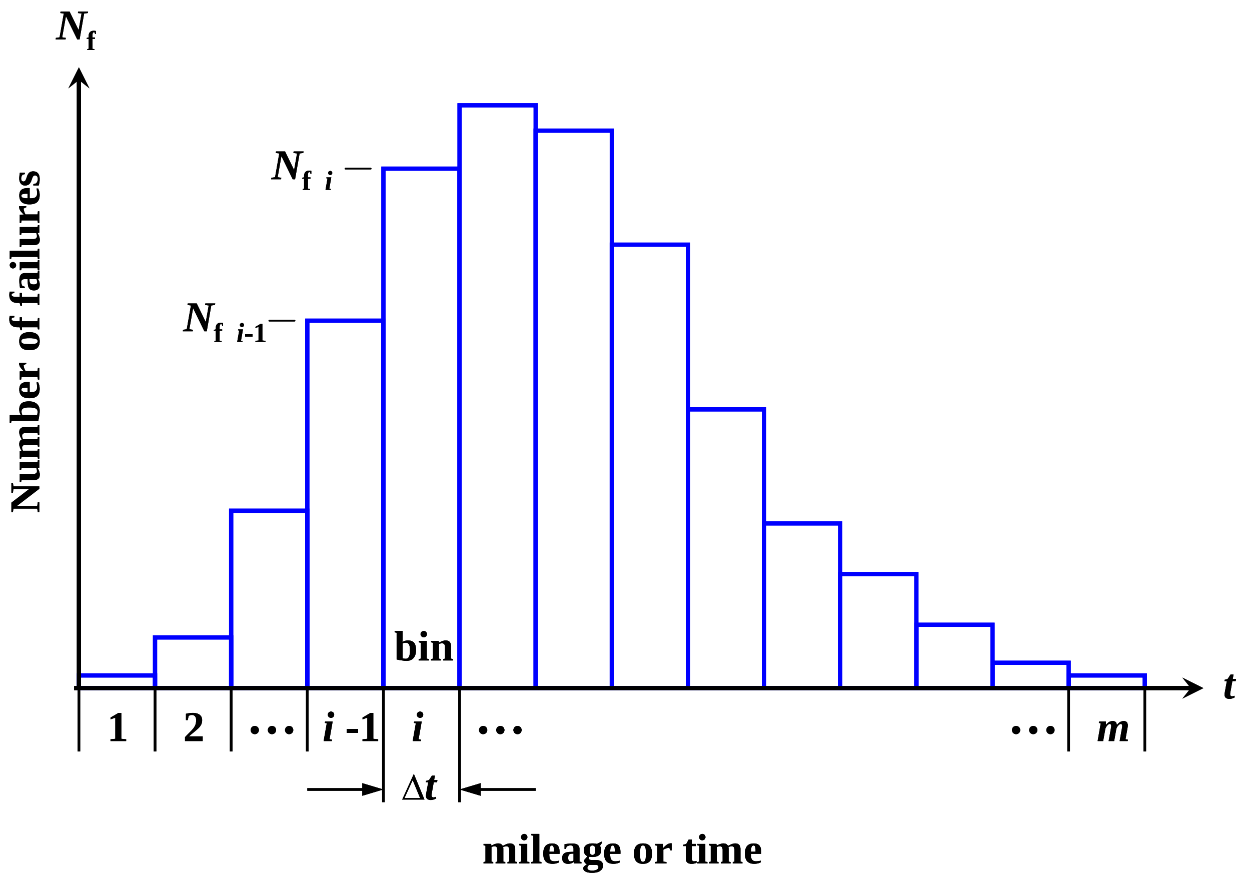
$$\Rightarrow \boxed{R(t) = \exp[-\int_0^t \lambda dt]}$$

All we need is the failure rate

Reliability of Non-Repairable Systems



Reliability of Non-Repairable Systems



$$N_f = \sum_{i=1}^{m} N_{f_i}$$

$$\lambda_i = \frac{f_i}{1-F_i} = \frac{f_i}{1-\sum_{j=1}^{i-1}\frac{N_{f_j}}{N_f}} = \frac{N_{f_i}}{\left(N_f - \sum_{j=1}^{i-1} N_{f_j}\right)\Delta t}$$

Reliability of Non-Repairable Systems

$$H_i = \sum_{j=1}^{i} \lambda_j \Delta t$$

$$R_i = 1 - F_i = 1 - \sum_{j=1}^{i-1} \frac{N_{\mathrm{f}_i}}{N_{\mathrm{f}}}$$

$$R_i = e^{-H_i}$$

Reliability Calculation

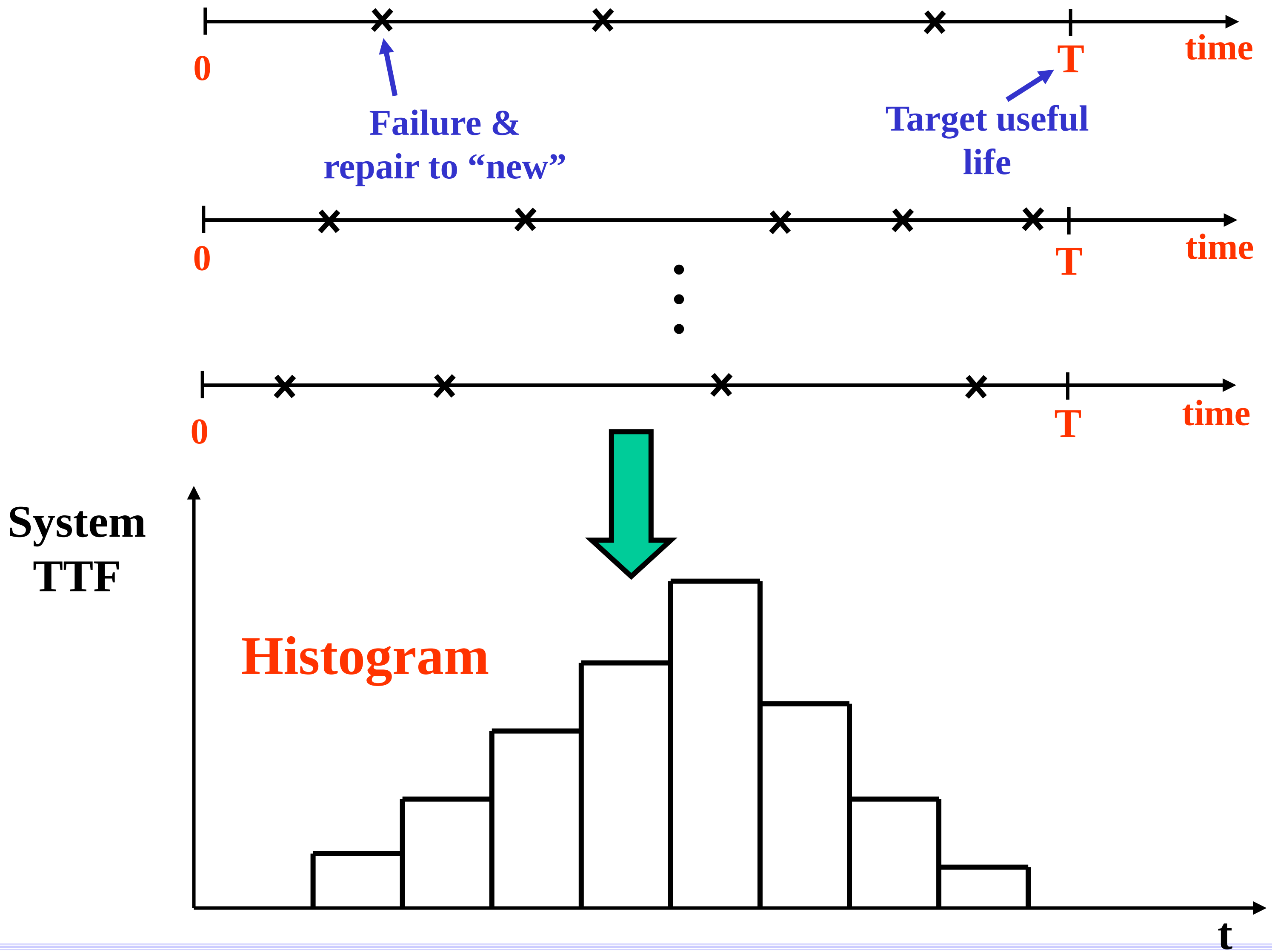
All we need for calculating the reliability of a system (non-repairable or repairable) is the system PDF of time to failure (TTF)

We use :

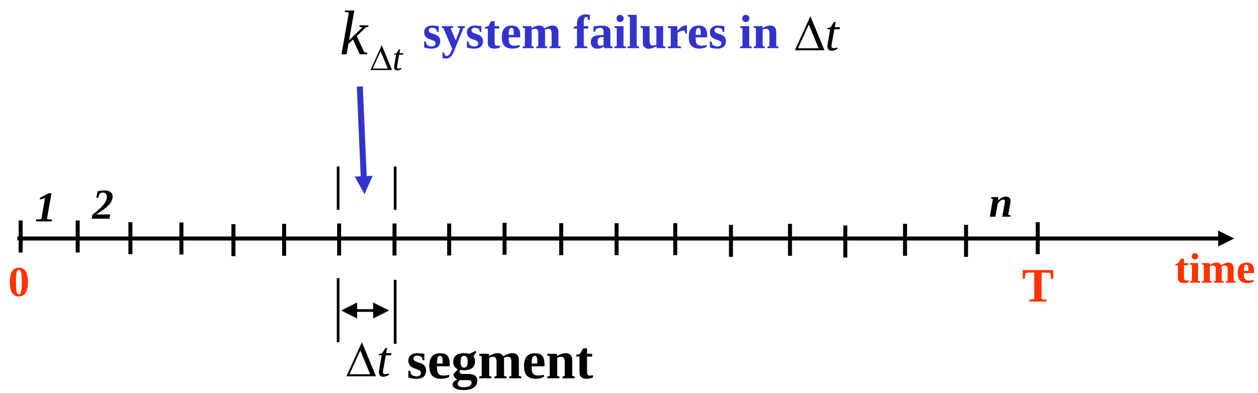
- **Data to estimate the PDF of TTF for each component**
- **Monte Carlo simulation to estimate the PDF of TTF for the system**

Basics of Reliability Methods (Repairable Systems)

Reliability of Repairable Systems



Reliability of Repairable Systems



$p = \frac{k_{\Delta t}}{N}$ **: Probability of failure in Δt (very small if $\Delta t \to 0$)**

# of systems (vehicles) in fleet

If p is independent of Δt segment $\Rightarrow$ **Homogeneous Poisson Process (HPP)**

If p depends on Δt segment $\Rightarrow$ **Non-Homogeneous Poisson Process (NHPP)**

Homogeneous Poisson Process (HPP)

pn : **Expected (average) # of failures / system in (0, T]**

Probability of failure in Δt

Define : $\boxed{\lambda T = pn}$

λ : **Average # of failures per system per unit time**

(failure rate, hazard rate, intensity rate, repair rate)

Homogeneous Poisson Process (HPP)

Reliability Calculation for HPP at $T = n\Delta t$

- **Assume statistically independent events at each** Δt

$$R(T) = (1-p)(1-p)\cdots(1-p) = (1-p)^n$$

$$= \left(1-\frac{\lambda t}{n}\right)\left(1-\frac{\lambda t}{n}\right)\cdots\left(1-\frac{\lambda t}{n}\right) = \left(1-\frac{\lambda t}{n}\right)^n = (1-\lambda\Delta t)^n$$

$= e^{-\lambda T}$ **if** $\Delta t \to 0$

Non-Homogeneous Poisson Process (NHPP)

Reliability Calculation for NHPP at $T = n\Delta t$

❖ **Assume statistically independent events at each** Δt

$$R(T) = (1 - p_1)(1 - p_2)\cdots(1 - p_n)$$

$$= (1 - \lambda_1 \Delta t)(1 - \lambda_2 \Delta t)\cdots(1 - \lambda_n \Delta t)$$

$$= \exp\left[-\int_0^T \lambda(t)\,dt \right] = e^{-H(T)} \quad \textbf{if} \quad \Delta t \to 0$$

Same formula with non-repairable systems

Cumulative Hazard Rate

Reliability Calculation

Series Systems

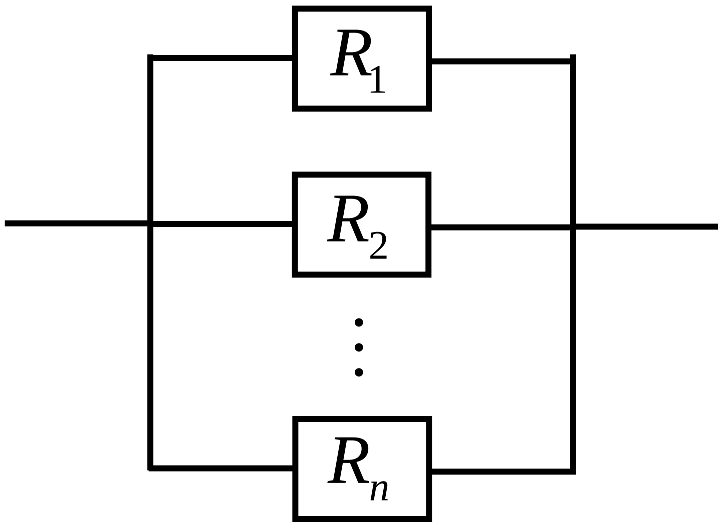
$$R = R_1 * R_2 * \cdots * R_n =$$

$$= e^{-\lambda_1 t} * e^{-\lambda_2 t} * \cdots * e^{-\lambda_n t} = e^{-\lambda t}$$

where : $$\boxed{\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_n}$$

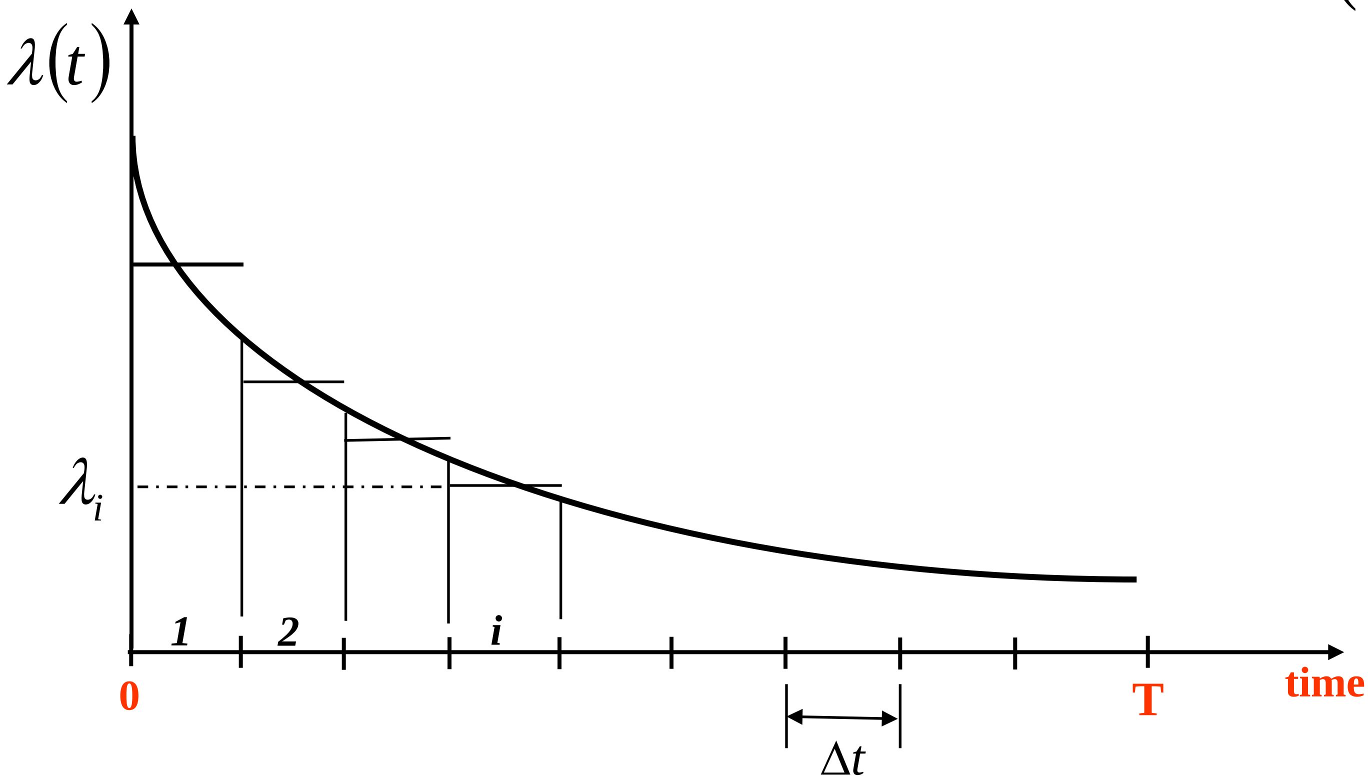
Reliability Calculation

Parallel Systems



$$R = 1 - (1 - R_1) * (1 - R_2) * \cdots * (1 - R_n)$$

Calculation of Hazard Rate $\lambda(t)$

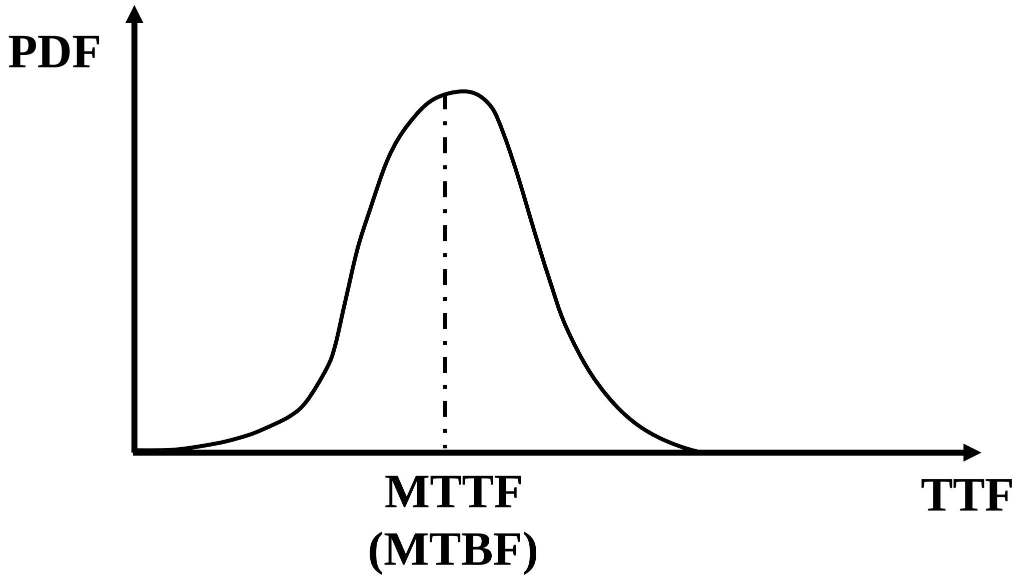


Determine average # of failures $k_{\Delta t}(t)$ among N systems in each Δt. Then :

$$\boxed{\lambda(t) = \frac{k_{\Delta t}(t)}{N\Delta t}}$$

Calculation of Hazard Rate $\lambda(t)$

For constant hazard rate systems : $\boxed{\lambda = \frac{1}{MTBF}}$



For most engineering systems, the hazard rate IS NOT constant. To estimate it, we need the PDF of TTF for each component.

Reliability Calculation

All we need for calculating the reliability of a system (non-repairable or repairable) is the system PDF of time to failure (TTF)

We use :

- **Data to estimate the PDF of TTF for each component**
- **Monte Carlo simulation to estimate the PDF of TTF for the system**

Estimation of the PDF (or CDF) of the TTF (TBF) using Limited, Censored Data

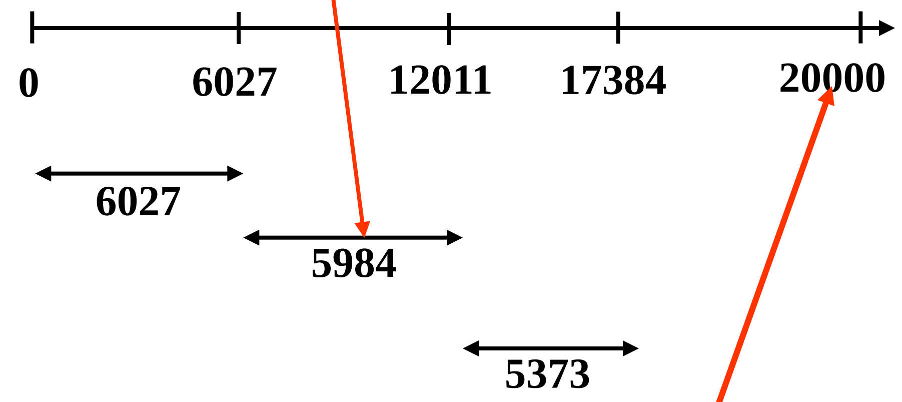
- **Censored MLE Approach**

Limited Data

Group L1

Original data Vehicle#	Original data mileage	Updated data Vehicle#	Updated data mileage
10	741	1	10247
4	5273	2	9044
7	6027	2	8977
5	7398	3	13984
6	7495	3	4064
2	9044	4	5273
1	10247	4	9747
8	12008	5	7398
7	12011	5	7611
9	12014	6	7495
10	12074	6	7516
3	13984	7	6027
5	15009	7	5984
6	15011	7	5373
4	15020	8	12008
7	17384	9	12014
2	18021	10	741
3	18048	10	11333

Time Between Failures (TBF)



Censoring Mileage

Censored MLE Approach

- Using available limited data (TBFs and censoring mileage), "estimate" PDF of TBF using a censored MLE approach.
- Tail sample the PDF of previous step to "enhance" the original limited data.
- Using "enhanced" data from previous step, "better estimate" the PDF of TBF using an uncensored MLE approach.
- Using the PDF of previous step, a Bootstrap approach estimates statistics of TBF (e.g. distribution of MTBF, distribution of TBF standard deviation, etc.)

Notation

Group L1

Original data Vehicle#	Original data mileage	Updated data Vehicle#	Updated data mileage
10	741	1	10247
4	5273	2	9044
7	6027	2	8977
5	7398	3	13984
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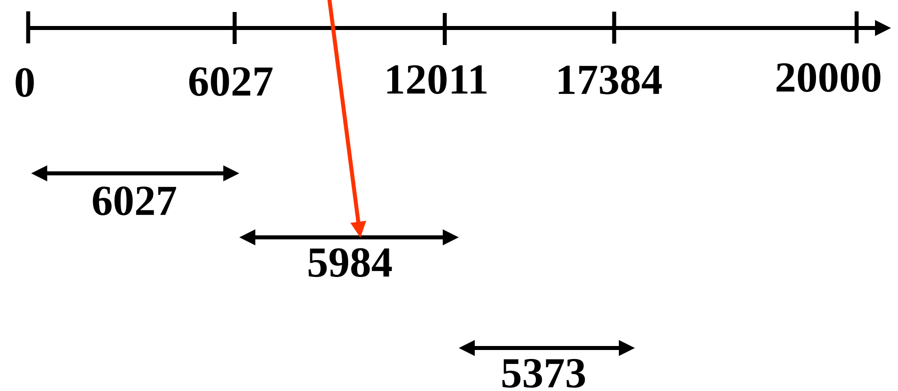
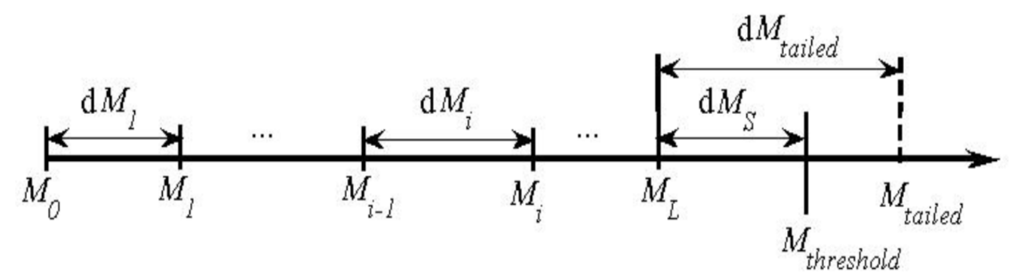
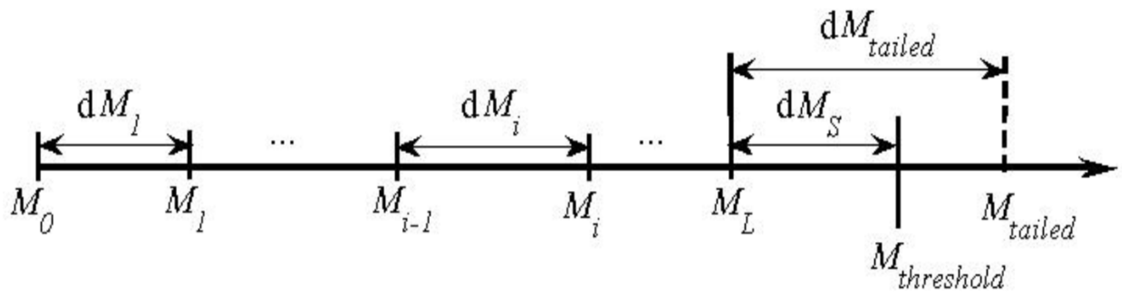
Time Between Failures (TBF)



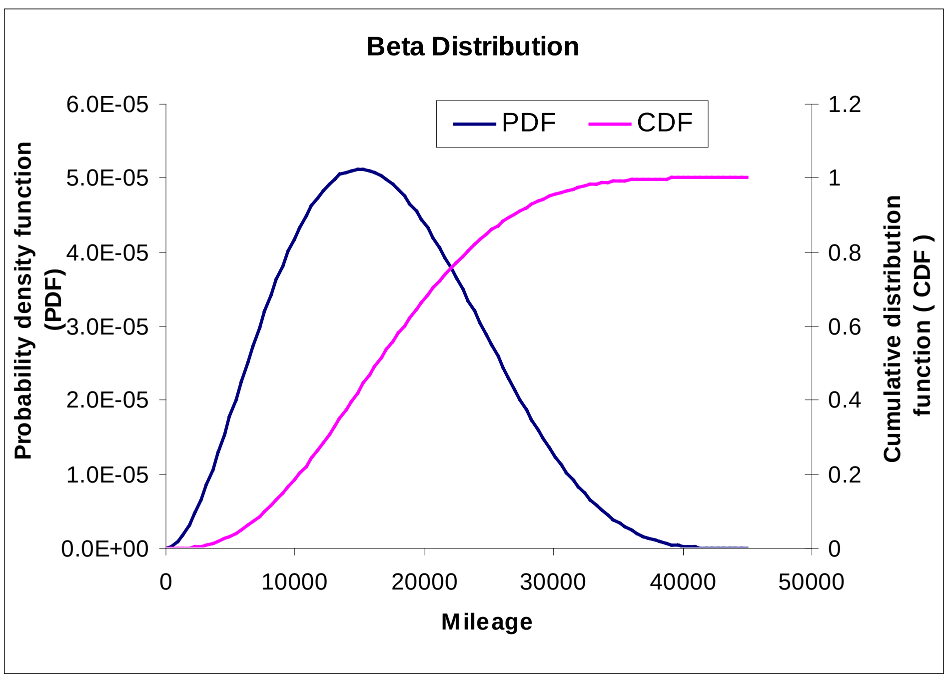


Observation / Assumption



$$\boxed{\mathrm{d}M_i = X_i \sim \beta(A, B, p, q), \quad (A \le X_i \le B, \text{ and } p > 0, q > 0)}$$

$$\boxed{f(x, A, B, p, q) = \beta(p,q)^{-1}(x - A)^{p-1}(B - x)^{q-1} / (B - A)^{p+q-1}, \quad (A \le x \le B, \; and \;\; p > 0, q > 0)}$$



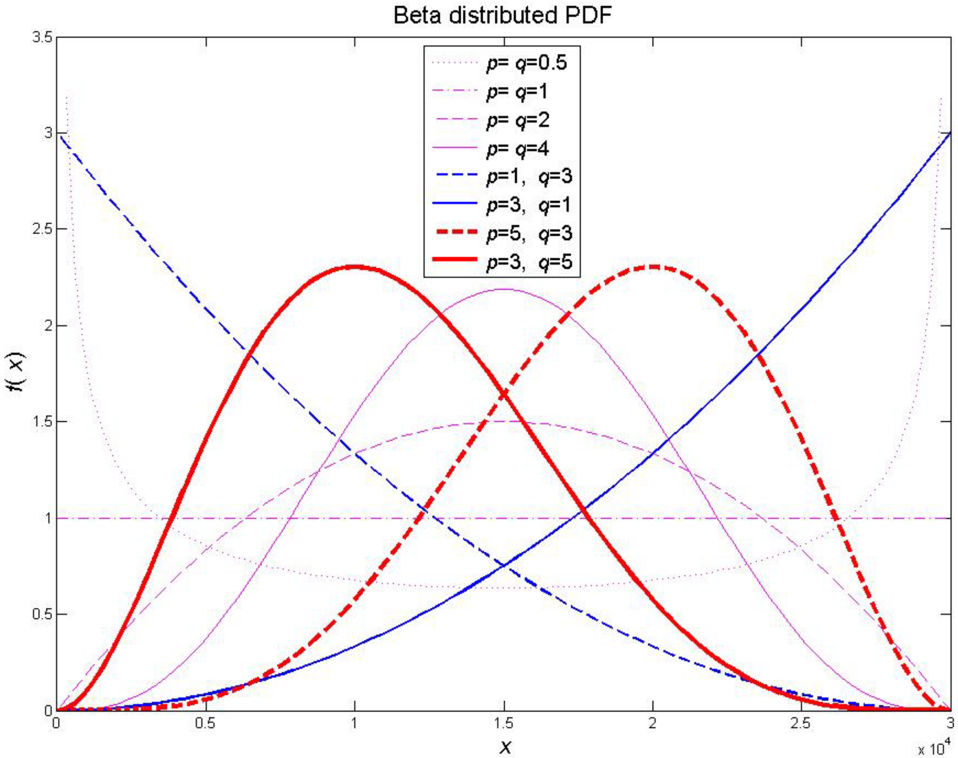
A = 0

B = 45,000 miles

p = 3, q = 5

Observation / Assumption

- **Beta distribution family is used to model TBF.**



$$A=0,\ B=30000$$

$$f(x,A,B,p,q)=\beta(p,q)^{-1}(x-A)^{p-1}(B-x)^{q-1}/(B-A)^{p+q-1},\quad (A\le x\le B,\text{ and } p>0,\ q>0)$$

MLE Approach

Determines parameters (*A, B, p, q*) of "most likely" Beta distribution using available data. It provides Likelihood function in Bayesian estimation.

Censored MLE

# of recorded failures

# of survivals

$$\underset{A,B,p,q}{Max} \prod_{i=1}^{N_F} f\left(x_i, A, B, p, q\right) \prod_{j=1}^{N_s} \left[1 - F\left(x_j, A, B, p, q\right)\right]$$

Beta PDF

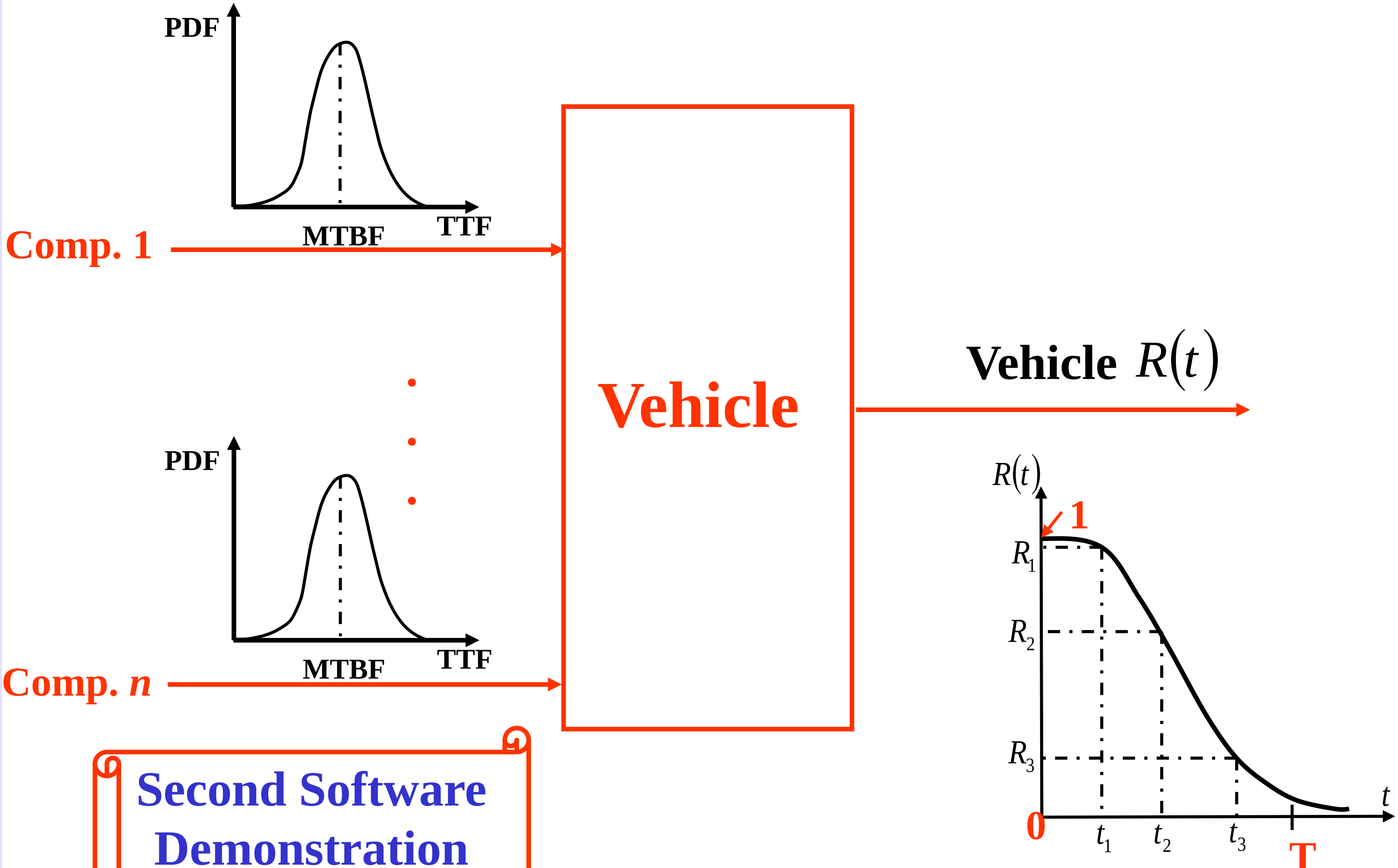
Beta CDF

Uncensored MLE

$$\underset{A,B,p,q}{Max} \prod_{i=1}^{N} f\left(x_i, A, B, p, q\right)$$

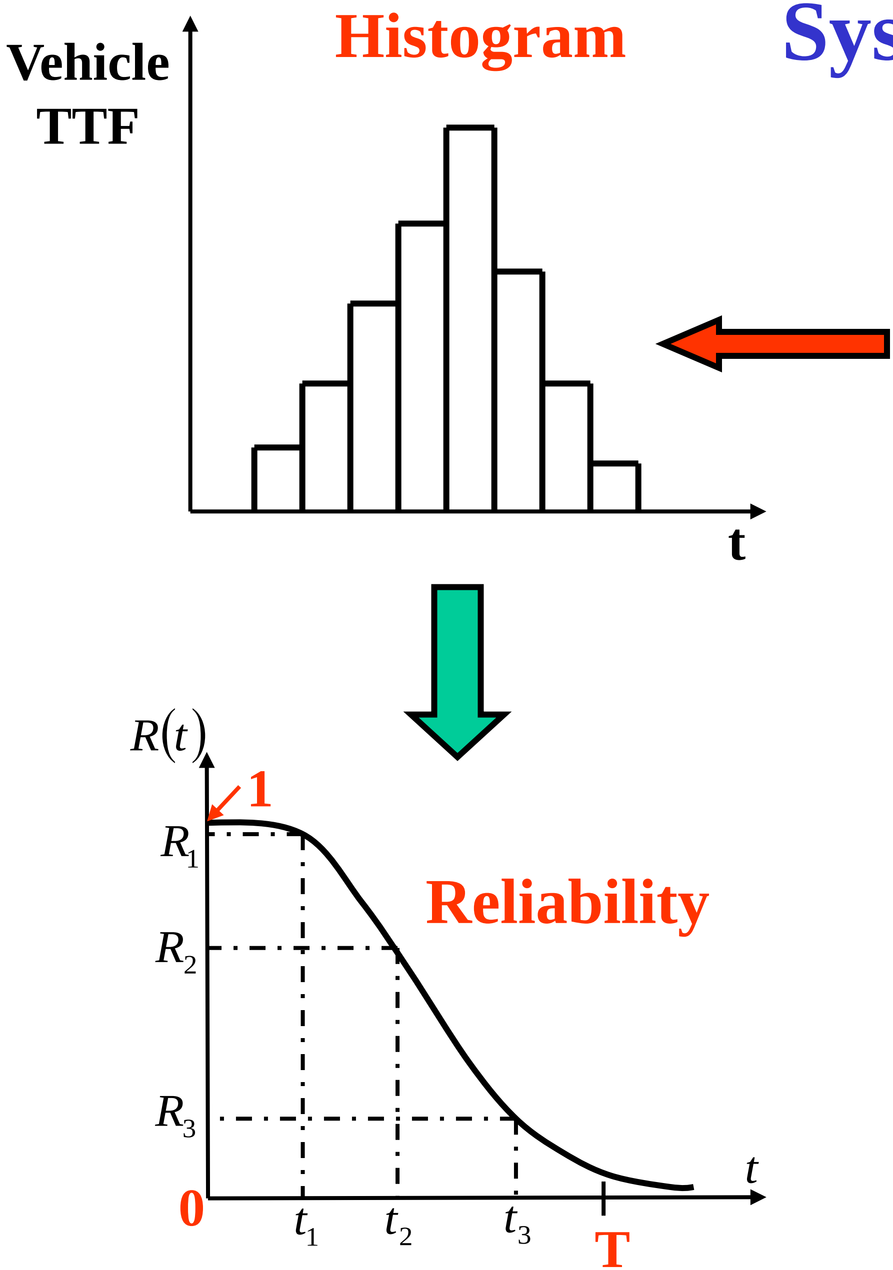
System Reliability and Reliability Allocation

System (Vehicle) Reliability



Second Software Demonstration

System (Vehicle) Reliability

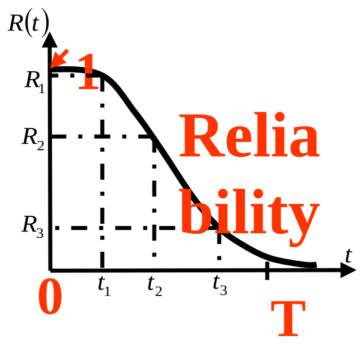


Monte Carlo Simulation

<u>For Vehicle :</u>

$$MTBF = \int_0^{\infty} R(t)\,dt$$

Reliability Allocation



Specify system (vehicle) reliability

Optimization

Determine required reliability of EACH component

This optimization problem DOES NOT have a unique solution

Reliability Allocation

One way to get a unique solution is to trade-off reliability and associated cost

$$\min_{\underline{R}_{comp}} \quad Cost$$

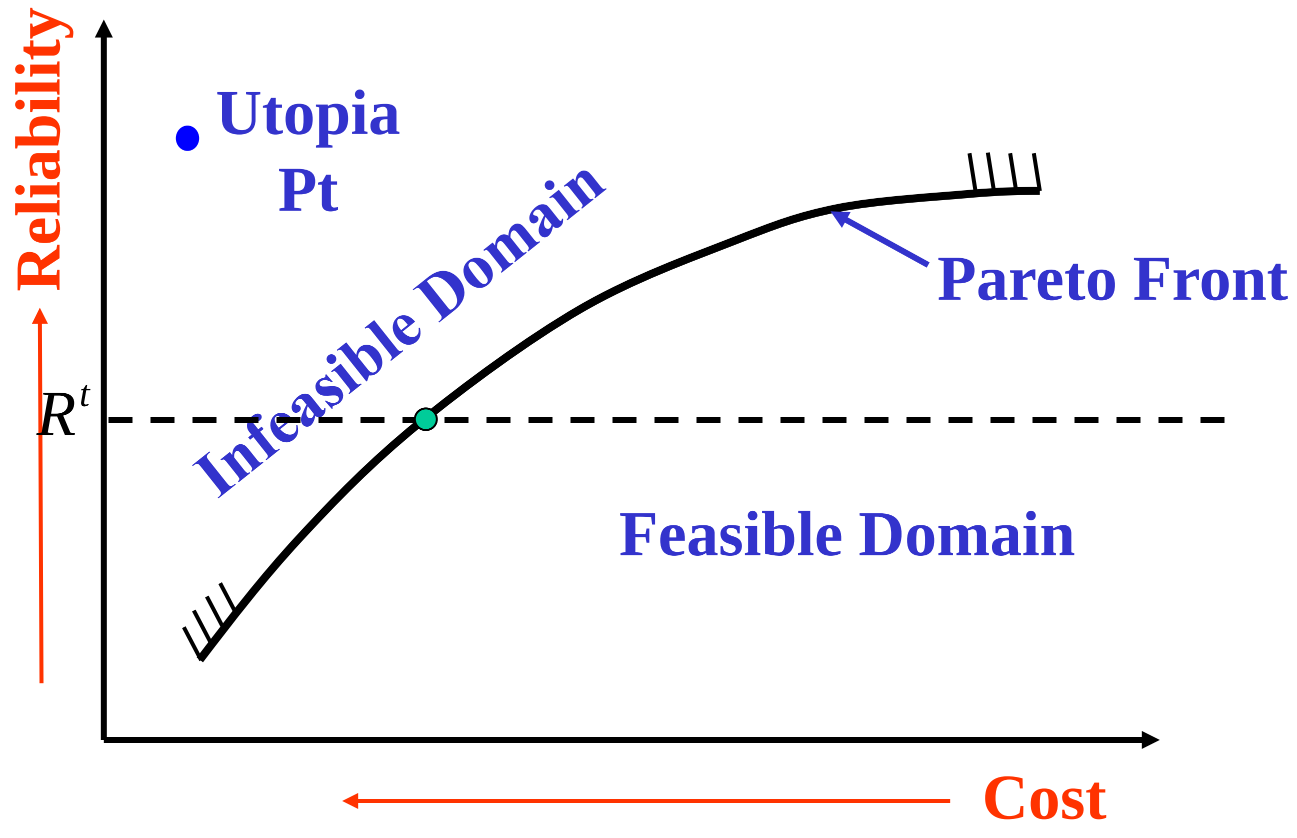
Target system reliability

$$\textbf{s. t.} \quad System \quad \mathrm{Re}\,liability = R^t$$

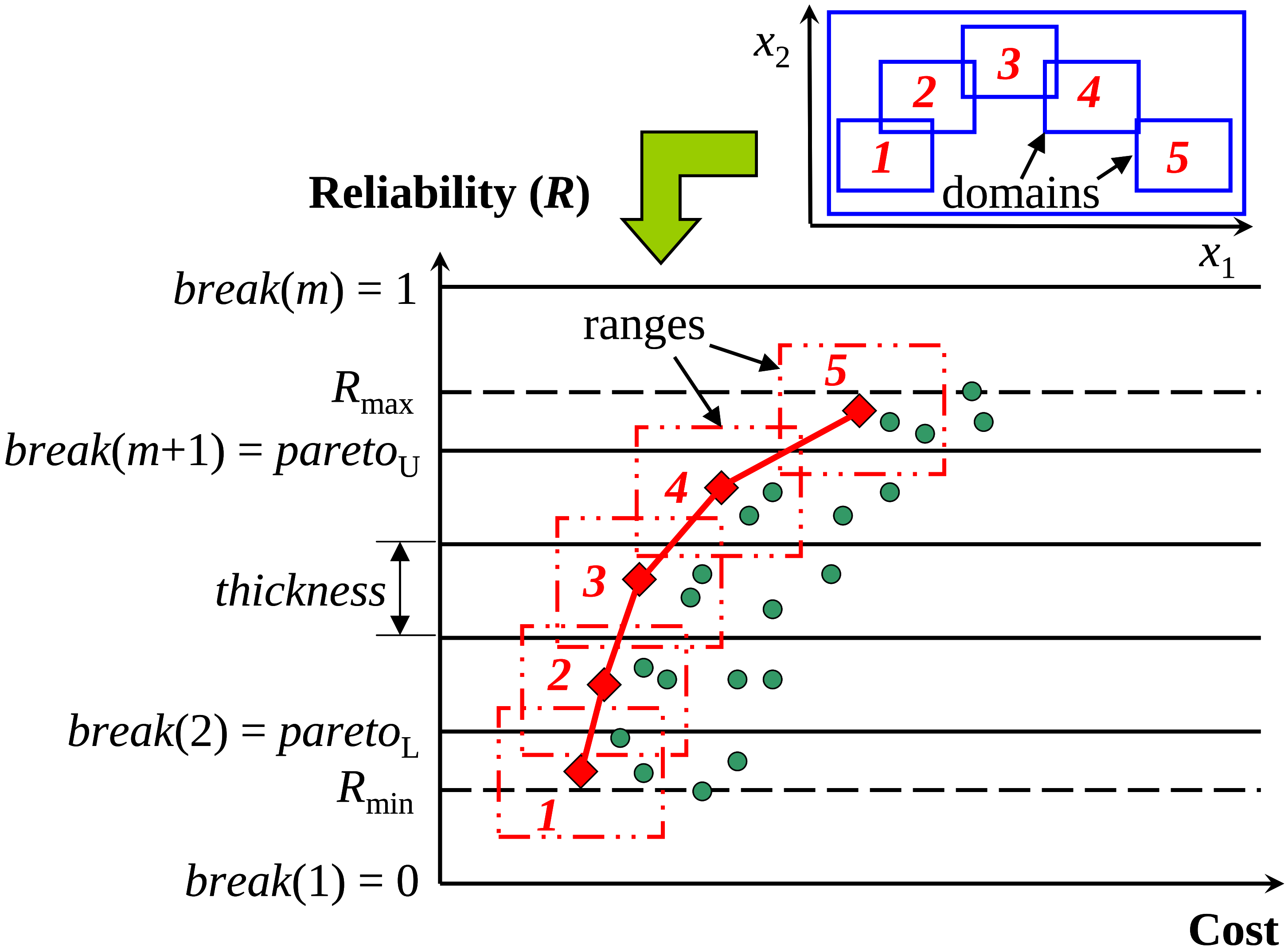
By varying R^t, we get the so called "Pareto Frontier."

Reliability vs Risk of Failure (Cost)

We want to maximize Reliability and simultaneously minimize Risk of failure (cost)



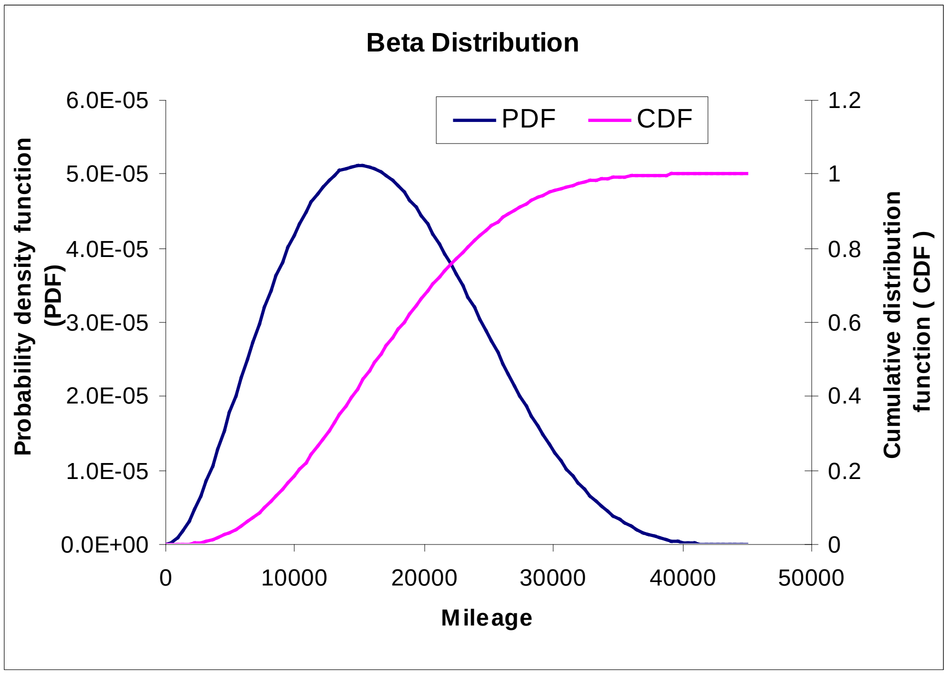
Reliability – Cost Pareto Front Calculation



$\overline{\sigma} = \frac{\sigma}{B - A}$

Definition of Design Variables

$$\boxed{f(x, A, B, p, q) = \beta(p,q)^{-1}(x - A)^{p-1}(B - x)^{q-1} / (B - A)^{p+q-1}, \quad (A \le x \le B, \; and \;\; p > 0, q > 0)}$$



$\mu = MTBF$

Assume constant COV

Then:

$$\overline{\mu} = \frac{\mu - A}{B - A} \qquad \overline{\sigma} = \frac{\sigma}{B - A}$$

$$p = \overline{\mu}\left(\frac{\overline{\mu}(1 - \overline{\mu})}{\overline{\sigma}^2} - 1\right),$$

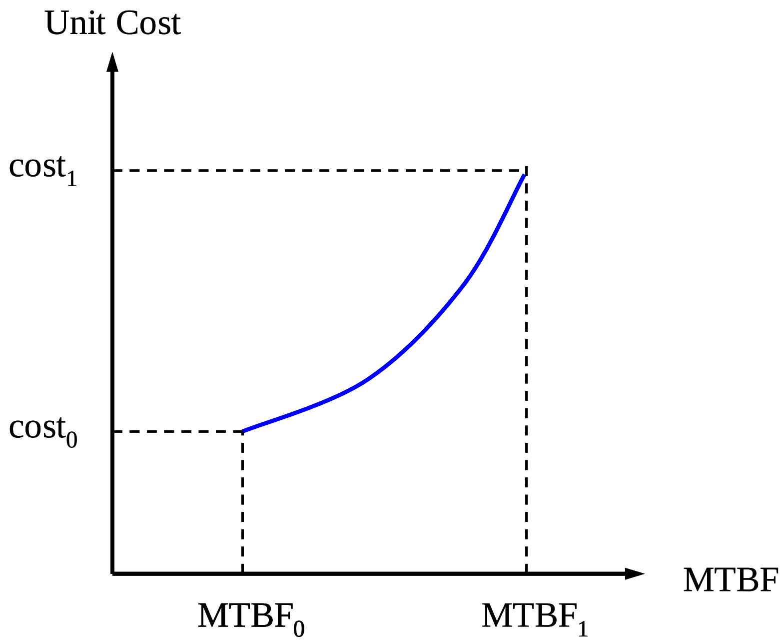
$$q = (1 - \overline{\mu})\left(\frac{\overline{\mu}(1 - \overline{\mu})}{\overline{\sigma}^2} - 1\right)$$

Reliability-Cost Relation

$$\text{cost} = \text{cost}_0\, e^{k(\text{MTBF}/\text{MTBF}_0 - 1)}$$ **: For each component**

$$\text{Cost} = \sum_{i_C=1}^{N_C} \left[\text{cost}_0\, e^{k(\text{MTBF}/\text{MTBF}_0 - 1)} (1 + \text{failure counts}) \right]_{i_C}$$

For system with Nc components

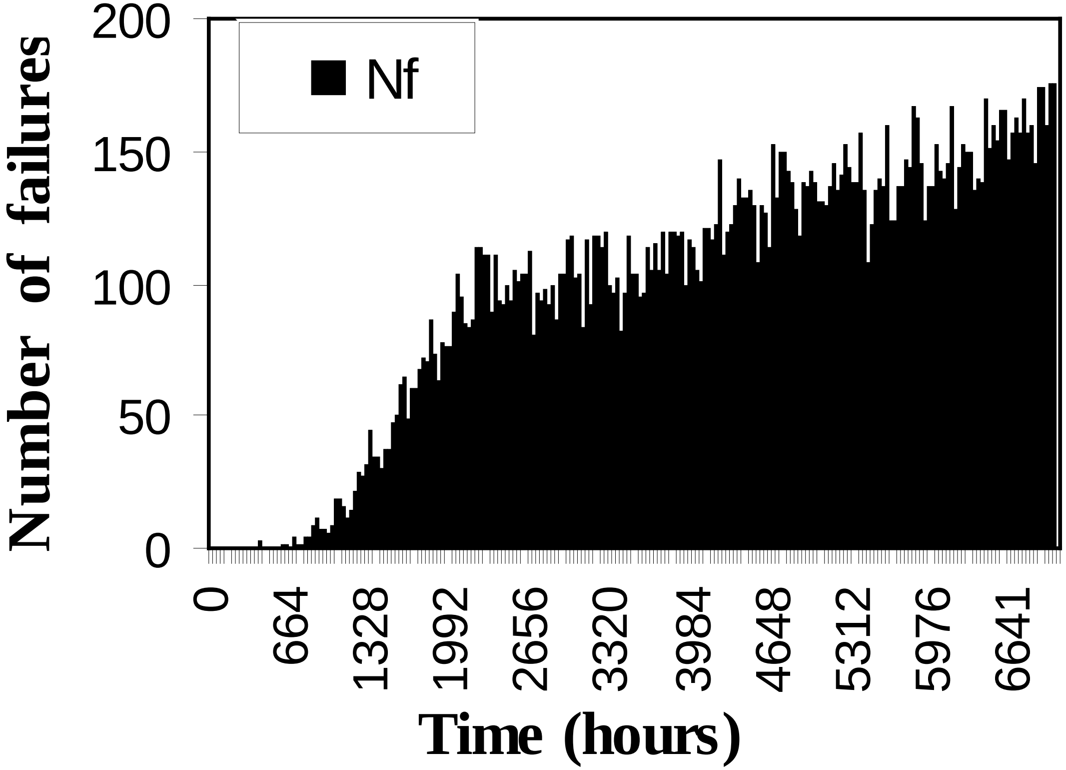


Example

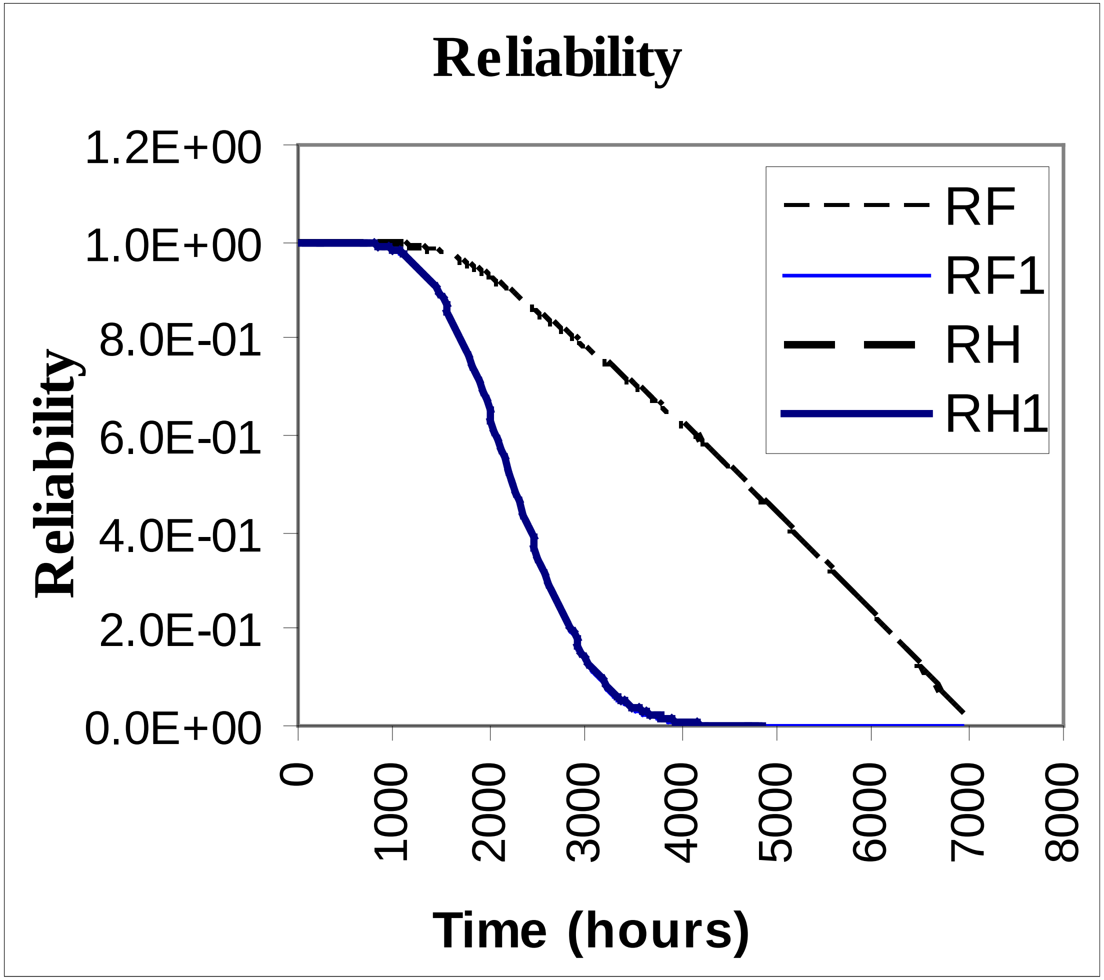
Input Information

Component Number Comp No.	*Baseline MTBF in hours ($MTBF_0$)*	*Coefficient of Variation*	B_{factor}	*Baseline cost ($Cost_0$)*	*k*
1	4076	0.3	3	$27,500.00	1
2	15000	0.3	3	$7,000.00	1
3	26510	0.3	3	$3,000.00	1
4	40000	0.3	3	$5,000.00	1
5	18000	0.3	3	$5,000.00	1
6	8000	0.3	3	$500.00	1
7	31809	0.3	3	$22,500.00	1
8	9520	0.3	3	$30,000.00	1
9	9713	0.3	3	$12,500.00	1
10	2330	0.3	3	$20,000.00	1
11	40000	0.3	3	$27,500.00	1
12	8614	0.3	3	$1,000.00	1
13	45000	0.3	3	$30,000.00	1
14	20000	0.3	3	$3,000.00	1
15	25000	0.3	3	$15,000.00	1

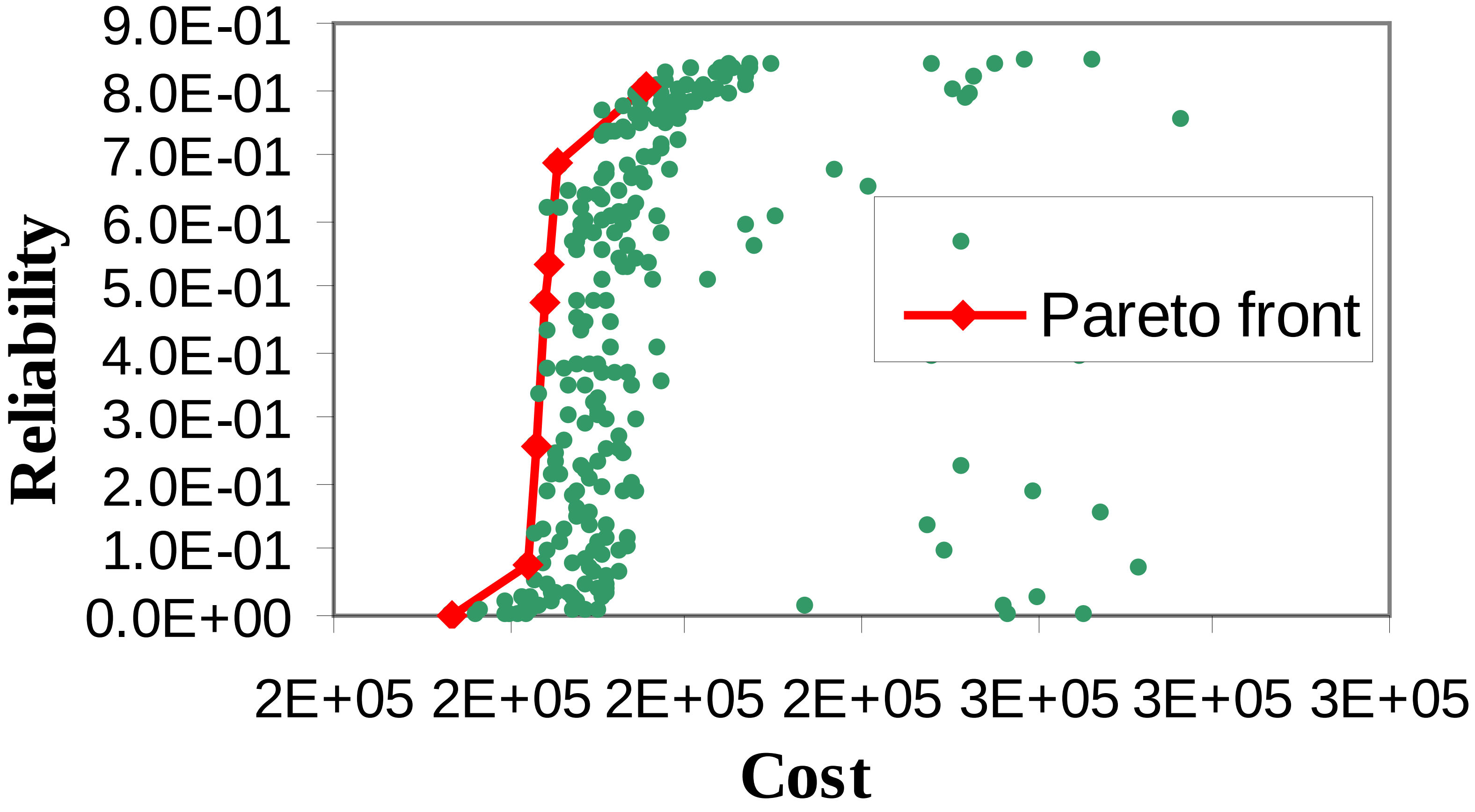
Histogram of System failures



Reliability Comparison between Repairable And Non-Repairable System



System Reliability-Cost Pareto Front



Q & A